

# Simple Scheme for Efficient Linear Optics Quantum Gates

T.C.Ralph, A.G.White, W.J.Munro and G.J.Milburn

*Centre for Quantum Computer Technology,*

*University of Queensland, QLD 4072, Australia*

*Fax: +61 7 3365 1242 Telephone: +61 7 3365 3412*

*email: [ralph@physics.uq.edu.au](mailto:ralph@physics.uq.edu.au)*

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## Abstract

We describe the construction of a conditional quantum control-not (CNOT) gate from linear optical elements following the program of Knill, Laflamme and Milburn [Nature **409**, 46 (2001)]. We show that the basic operation of this gate can be tested using current technology. We then simplify the scheme significantly.

## I. INTRODUCTION

Optics would seem to be a strong contender for realizing quantum computation circuits. Photons are easily manipulated and, as the electro-magnetic environment at optical frequencies can be regarded as vacuum, are relatively decoherence free. Indeed one of the earliest proposals [1] for implementing quantum computation was based on encoding each qubit in two optical modes, each containing exactly one photon. Unfortunately, 2 qubit gates require strong interactions between single photons. Such interactions would require massive, reversible non-linearities well beyond those presently available.

Recently Knill, Laflamme and Milburn (KLM) found a way to circumvent this problem and implement efficient quantum computation using only passive linear optics, photodetectors, and single photon sources [2]. This efficient linear optical quantum computing (ELOQC) is distinct from other linear optical schemes [3] which are not efficiently scalable.

Although containing only linear elements, the optical networks described by KLM are complex and would present major stability and mode matching problems in their construction. There is thus considerable interest in finding the simplest physical implementations of the KLM techniques. In this manuscript we investigate this problem and find a major simplification of the original proposal.

We begin by reviewing the technique via which non-deterministic gates can be used to implement an efficiently scalable system and in Section 3 the physics of a basic non-deterministic gate, the NS gate, is discussed. In Section 4 we describe the construction of a non-deterministic quantum CNOT gate using two NS gates. Full scalability of this gate requires high efficiency, 0, 1, 2 photon discriminating photon counters. Such detectors presently only exist in prototype form [4]. However, in Section 5 we show that the basic

operation of this circuit can be tested with current detector technology. We then describe the simplified gate.

A non-deterministic CNOT gate with a simple linear architecture, but requiring triggered entangled sources as a resource, has been suggested recently [5]. In contrast our scheme requires only separable input states. Also recently proposed is a linear optical scheme for the probabilistic purification of non-maximal polarization entangled states [6]. Although the linear elements play the role of CNOT gates in this protocol, they do not exhibit the full CNOT logic of the gates described here.

## II. GATE OPERATION VIA TELEPORTATION

Arbitrary quantum gate operations can be implemented if one has the ability to implement arbitrary single qubit rotations and two qubit CNOT gates. Single qubit operations can easily be implemented with single photons and a non-deterministic CNOT gate is described in this manuscript. However a cascaded sequence of such non-deterministic gates would be useless for quantum computation because the probability of many gates working in sequence decreases exponentially. This problem may be avoided by using a teleportation protocol [7] to implement quantum gates. The idea that teleportation can be used for universal quantum computation was first proposed by Gottesman and Chuang [8].

A teleportation circuit is represented in Fig.1(a). A qubit in an unknown state  $|\alpha\rangle$  is teleported by making a joint Bell measurement ( $B$ ) of it and one half of a Bell pair  $|\Phi\rangle$ . Depending on the result of the measurements,  $\sigma_x$  and  $\sigma_z$  manipulations are made on the other half of the Bell pair resulting in accurate reconstruction of the unknown state. A key issue is that the Bell pair plays the role of a resource in the protocol. That is, it can be prepared “off-line” and then used when necessary to teleport the qubit. Now consider the quantum circuit shown in Fig.1(b). Two unknown qubits are individually teleported and then a CNOT gate is implemented. Obviously, but not very usefully, the result is CNOT operation between the input and output qubits. However, as shown in Ref. [8], the commutation relations between CNOT and  $\sigma_x$  and  $\sigma_z$  are quite simple, such that the circuits of Fig.1(b) and 1(c) are in fact equivalent. But in the circuit of Fig.1(c) the problem of implementing a CNOT gate has been reduced to that of producing the required entanglement resource. The entanglement resource required could be produced from separable input states using three CNOT gates: one each to produce the Bell pairs plus the one shown in Fig.1(c). But the point is that these need not be deterministic gates. Non-deterministic CNOT gates could be used in a trial and error manner to build up the necessary resource off-line. It could then be used when required to implement the gate.

A remaining issue is the performance of the Bell measurements required in the teleportation protocol. These cannot be performed exactly with linear optics. KLM showed that by using the appropriate entangled resource the teleportation step can be made near deterministic. The near deterministic teleportation protocol requires only linear optics, photon counting and fast feedforward, albeit with a significant resource overhead. Alternatively, progress has recently been made towards implementing Bell measurements using non-linear optics [9].

### III. THE NS GATE

The basic element in the construction of our non-deterministic CNOT gate is the non-linear sign-shift (NS) gate [2]. This is a non-deterministic gate the operation of which is conditioned on the detection of an auxiliary photon. When successful the gate implements the following transformation on signal state  $|\psi\rangle$

$$|\psi\rangle = \alpha|0\rangle + \beta|1\rangle + \gamma|2\rangle \rightarrow |\psi'\rangle = 0.5(\alpha|0\rangle + \beta|1\rangle - \gamma|2\rangle) \quad (1)$$

where the lack of normalization of the transformed state reflects the fact that the gate has a probability of success of  $0.25 = (0.5)^2$ .

Fig.2 shows a realization of this gate. Two ancilla modes are required. A single photon is injected into one of the ancilla and the other is unoccupied. The first, second and third beamsplitters have intensity reflectivities  $\eta_1$ ,  $\eta_2$  and  $\eta_3$  respectively. The beamsplitters are phase asymmetric: transmission from either side and reflection off the “black” surface of these beamsplitters results in no phase change, whilst reflection off the “grey” surface results in a sign change. When a single photon is counted at the “1” ancilla output and no photon is counted at the “0” ancilla output (as indicated in the figure) the transformation of Eq.1 is implemented if a suitable choice of beamsplitter reflectivities is made. Let us see how this works.

Suppose first that the signal mode is in the vacuum state, i.e.  $|\psi\rangle = |0\rangle$ . The probability amplitude,  $C$ , for the ancilla photon to appear at the “1” output port is given by

$$C = \sqrt{\eta_1\eta_2\eta_3} + \sqrt{(1-\eta_1)(1-\eta_3)} \quad (2)$$

Now suppose the input is a single photon state, i.e.  $|\psi\rangle = |1\rangle$ . If a photon arrives at the “1” output port and no photon arrives at the “0” port then a single photon must have exited the signal output. We wish the probability amplitude for this event to also be  $C$ . This means

$$\begin{aligned} C &= \sqrt{\eta_1\eta_3}(1-\eta_2) - (\sqrt{\eta_1\eta_2\eta_3} + \sqrt{(1-\eta_1)(1-\eta_3)})\sqrt{\eta_2} \\ &= \sqrt{\eta_1\eta_3}(1-\eta_2) - C\sqrt{\eta_2} \end{aligned} \quad (3)$$

and thus

$$C = \frac{\sqrt{\eta_1\eta_3}(1-\eta_2)}{1 + \sqrt{\eta_2}} \quad (4)$$

Finally we consider the situation of a two photon input, i.e.  $|\psi\rangle = |2\rangle$ . If a single photon arrives at the “1” port and no photon arrives at the “0” port then two photons must have exited at the signal output. To obtain the sign change of Eq.1 we require the probability amplitude for this event to be  $-C$ . This means

$$\begin{aligned} -C &= -\sqrt{\eta_1\eta_3\eta_2}(1-\eta_2) - \sqrt{\eta_2}(\sqrt{\eta_1\eta_3}(1-\eta_2) - (\sqrt{\eta_1\eta_2\eta_3} + \sqrt{(1-\eta_1)(1-\eta_3)})\sqrt{\eta_2}) \\ &= \eta_2 C - 2\sqrt{\eta_1\eta_2\eta_3}(1-\eta_2) \end{aligned} \quad (5)$$

Substituting Eq.4 into Eq.5 gives the result

$$\eta_2 = (\sqrt{2} - 1)^2 \quad (6)$$

Substituting back into Eq.4 and Eq.2 we can solve for  $\eta_1$ ,  $\eta_3$  and  $C$ . The maximum value for  $C$  is achieved when

$$\eta_1 = \eta_3 = \frac{1}{(4 - 2\sqrt{2})} \quad (7)$$

and is

$$C = 0.5 \quad (8)$$

Thus the transformation of Eq.1 is implemented whenever a single photon is recorded at port “1” and no photon is found at port “0”. On average this will occur 25% of the time since  $|C|^2 = 0.25$ .

#### IV. THE CNOT GATE

A conditional CNOT gate can now be implemented using two NS gates. The layout for doing this is shown schematically in Fig. 3. We employ dual rail logic such that the “control in” qubit is represented by the two bosonic mode operators  $c_H$  and  $c_V$ . A single photon occupation of  $c_H$  with  $c_V$  in a vacuum state will be our logical 0, which we will write  $|H\rangle$  (to avoid confusion with the vacuum state). Whilst a single photon occupation of  $c_V$  with  $c_H$  in a vacuum state will be our logical 1, which we will write  $|V\rangle$ . Of course superposition states can also be formed. Similarly the “target in” is represented by the bosonic mode operators  $t_H$  and  $t_V$  with the same interpretations as for the control. The beamsplitters,  $B1$ ,  $B2$ ,  $B3$  and  $B4$  are all 50:50.

The four modes  $c_H$ ,  $c_V$ ,  $t_H$  and  $t_V$  are all the same polarization. The use of the “H”, “V” nomenclature alludes to the standard situation in which the two modes of the dual rail logic are orthogonal polarization modes. Conversion of a polarization qubit into the spatial encoding used to implement the CNOT gate can be achieved experimentally by passing the the photon through a polarizing beamsplitter, to spatially separate the modes, and then using a half-wave plate to rotate one of the modes into the same polarization as the other. After the gate, the reverse process can be used to return the encoding to polarization.

The layout of Fig. 4 contains two nested, balanced Mach-Zehnder interferometers. The target modes are combined and then re-separated forming the “T” interferometer. One arm of the T interferometer and the  $c_V$  mode of the control are combined to form another interferometer, the “C” interferometer. NS gates are placed in both arms of the C interferometer. The essential feature of the system is that if the control photon is in the  $c_H$  mode then there is never more than one photon in the C interferometer, so the NS gates do not produce a change, the T interferometer remains balanced and the target qubits exit in the same spatial modes in which they entered. On the other hand if the control photon is in mode  $c_V$  then there is a two photon component in the C interferometer which suffers a sign change due to the NS gates. This leads to a sign change in one arm of the T interferometer and the target qubit exits from the opposite mode from which it entered.

Let us consider the systems operation in more detail: If the control is in a logical 0 then the mode  $c_V$  will be in a vacuum state. Consider the line labeled  $x$  in Fig.3 lying just before the NS gates. The state of the system at this point is given by

$$|\psi\rangle_x = \frac{1}{\sqrt{2}}|1001\rangle \pm \frac{1}{2}(|1100\rangle - |1010\rangle) \quad (9)$$

where the left to right ordering is equivalent to the top to bottom ordering in Fig.3. The  $+$  occurs when the target input state is  $|H\rangle$ , the  $-$  occurs when the target input state is  $|V\rangle$ . Now consider the state of the system directly after the NS gates operate on the middle two modes (indicated by the line  $y$  in Fig.3). Substituting from Eq.1 we find  $|\psi\rangle_y = 0.25|\psi\rangle_x$ . That is the gates do not effect the states in the arms of the C interferometer (conditional on the detection of photons at the “1” ports of the NS gates). As both interferometers are balanced they will just return the same outputs as they had inputs. Thus  $c_{Vo}$  will be a vacuum mode, and if the target input photon was in  $t_H$ , it will emerge in  $t_{Ho}$ ; or if it was in  $t_V$ , it will emerge in  $t_{Vo}$ . In other words the control and target qubits will remain in the same states.

On the other hand if the control is in a logical 1, then the  $c_V$  mode will contain one photon. The state at  $x$  is now

$$|\psi\rangle_x = \frac{1}{2}(|0101\rangle + |0011\rangle \pm (|0200\rangle - |0020\rangle)) \quad (10)$$

The two photon amplitudes suffer sign changes (conditional on the detection of photons at the “1” ports of the NS gates) such that the state at  $y$ , after the NS gates, is now

$$|\psi\rangle_y = 0.25\left(\frac{1}{2}(|0101\rangle + |0011\rangle \mp (|0200\rangle - |0020\rangle))\right) \quad (11)$$

This leads to a sign change in the returning beam of the T interferometer which in turn results in a swap between the inputs and outputs of the T interferometer. Thus if the target input photon was in  $t_H$  it will emerge in  $t_{Vo}$  or if it was in  $t_V$  it will emerge in  $t_{Ho}$ . The control output,  $c_{Vo}$  also suffers a sign change, but this does not change its logical status. In other words the control is unchanged but the target qubit will change states.

The truth table of the device is thus

$$\begin{aligned} |H\rangle_c|H\rangle_t &\rightarrow |H\rangle_c|H\rangle_t, & |H\rangle_c|V\rangle_t &\rightarrow |H\rangle_c|V\rangle_t \\ |V\rangle_c|H\rangle_t &\rightarrow |V\rangle_c|V\rangle_t, & |V\rangle_c|V\rangle_t &\rightarrow |V\rangle_c|H\rangle_t \end{aligned} \quad (12)$$

Which is CNOT logic.

It is useful to also look at this arrangement in the Heisenberg picture. Referring again to Fig.3 our input modes are  $c_H$  and  $c_V$  for the control,  $t_H$  and  $t_V$  for the target, and the ancilla modes  $a_1$ ,  $a_2$ ,  $v_1$  and  $v_2$ . The initial state of  $c_i$ ,  $t_j$ ,  $a_1$ ,  $a_2$  is  $|1, 1, 1, 1\rangle$  where  $i, j = H$  or  $V$ . The other modes are initially in the vacuum state  $|0, 0, 0, 0\rangle$ . We propagate these modes through the system and obtain the following expressions for the output modes

$$\begin{aligned} c_{Ho} &= c_H & c_{Vo} &= \frac{1}{\sqrt{2}}(d'_1 + d'_2) \\ t_{Ho} &= \frac{1}{\sqrt{2}}(t'' + t''') & t_{Vo} &= \frac{1}{\sqrt{2}}(t'' - t''') \\ a_{1o} &= \sqrt{\eta_3}a''_1 + \sqrt{1 - \eta_3}v'_1 & a_{2o} &= \sqrt{\eta_3}a''_2 + \sqrt{1 - \eta_3}v'_2 \end{aligned} \quad (13)$$

where

$$\begin{aligned}
t'' &= \frac{1}{\sqrt{2}}(d'_1 - d'_2) & t''' &= \frac{1}{\sqrt{2}}(t_H - t_V) \\
t' &= \frac{1}{\sqrt{2}}(t_H + t_V) & a''_1 &= \sqrt{\eta_2}a'_1 + \sqrt{1 - \eta_2}d_1 \\
a''_2 &= \sqrt{\eta_2}a'_2 + \sqrt{1 - \eta_2}d_2 & a'_1 &= \sqrt{\eta_1}a_1 + \sqrt{1 - \eta_1}v_1 \\
a'_2 &= \sqrt{\eta_1}a_2 + \sqrt{1 - \eta_1}v_2 & v'_1 &= \sqrt{1 - \eta_1}a_1 - \sqrt{\eta_1}v_1 \\
v'_2 &= \sqrt{1 - \eta_1}a_2 - \sqrt{\eta_1}v_2 & d'_1 &= \sqrt{1 - \eta_2}a'_1 - \sqrt{\eta_2}d_1 \\
d'_2 &= \sqrt{1 - \eta_2}a'_2 - \sqrt{\eta_2}d_2 & d_1 &= \frac{1}{\sqrt{2}}(c_V + t') \\
d_2 &= \frac{1}{\sqrt{2}}(c_V - t') & &
\end{aligned} \tag{14}$$

The logical statements of Eq.12 can then be realized through measurements of 4-fold coincidences. Thus if the initial state is  $|H\rangle_c|H\rangle_t$  then we find

$$\begin{aligned}
\langle c_{Ho}^\dagger c_{Ho} \ t_{Ho}^\dagger t_{Ho} \ a_{1o}^\dagger a_{1o} \ a_{2o}^\dagger a_{2o} \rangle &= \frac{1}{16} \\
\langle c_{Ho}^\dagger c_{Ho} \ t_{Vo}^\dagger t_{Vo} \ a_{1o}^\dagger a_{1o} \ a_{2o}^\dagger a_{2o} \rangle &= 0 \\
\langle c_{Vo}^\dagger c_{Vo} \ t_{Vo}^\dagger t_{Vo} \ a_{1o}^\dagger a_{1o} \ a_{2o}^\dagger a_{2o} \rangle &= 0 \\
\langle c_{Vo}^\dagger c_{Vo} \ t_{Ho}^\dagger t_{Ho} \ a_{1o}^\dagger a_{1o} \ a_{2o}^\dagger a_{2o} \rangle &= 0
\end{aligned} \tag{15}$$

and similarly for the initial state  $|H\rangle_c|V\rangle_t$  we find

$$\langle c_{Ho}^\dagger c_{Ho} \ t_{Vo}^\dagger t_{Vo} \ a_{1o}^\dagger a_{1o} \ a_{2o}^\dagger a_{2o} \rangle = \frac{1}{16} \tag{16}$$

with all other moments zero. However for initial state the  $|V\rangle_c|H\rangle_t$  we find

$$\langle c_{Vo}^\dagger c_{Vo} \ t_{Vo}^\dagger t_{Vo} \ a_{1o}^\dagger a_{1o} \ a_{2o}^\dagger a_{2o} \rangle = \frac{1}{16} \tag{17}$$

with the other moments zero and for the initial state  $|V\rangle_c|V\rangle_t$  we find

$$\langle c_{Vo}^\dagger c_{Vo} \ t_{Ho}^\dagger t_{Ho} \ a_{1o}^\dagger a_{1o} \ a_{2o}^\dagger a_{2o} \rangle = \frac{1}{16} \tag{18}$$

with the other moments zero. As expected the factor  $1/16$  appears as we have employed two NS gates each of which works on average 25% of the time. It can also be verified that injection of the control qubit in the superposition states  $(1/\sqrt{2})(|H\rangle \pm |V\rangle)$  with the target in  $|H\rangle$  or  $|V\rangle$  produces correlations corresponding to the 4 entangled Bell states, as expected from quantum CNOT operation.

## V. SIMPLIFIED GATE OPERATION

A major experimental advantage to this set-up, as compared to the test circuit suggested in Ref. [2], is that we can work in the coincidence basis. This allows low efficiency detectors

and spontaneous single photon sources to be used to demonstrate the basic operation of the gate. Of course incorporating these gates in a scalable system as discussed in Section II requires one to know that the gate has successfully operated without destroying the output. It is straightforward to show from Eqs.13 that detection of one and only one photon in modes  $a_{1o}$  and  $a_{2o}$  and no photons in modes  $v_{1o}$  and  $v_{2o}$  is sufficient to ensure successful operation of the gate without disturbing the control and target outputs. However low-loss, 0, 1, 2-photon discriminating detection would be needed to operate in this way.

Even in the coincidence basis the above implementation represents a major technological challenge. Four nested interferometers must simultaneously be mode matched and locked to sub-wavelength accuracy over the operation time of the gate. A major simplification is achieved by operating the NS gates in a biased mode. The idea is to set the reflectivities  $\eta_1$  and  $\eta_3$  in the NS gates to one, i.e. totally reflective. This removes the interferometers from both the NS gates, greatly reducing the complexity of the gate. Summing over the paths as before we find that the NS operation becomes

$$|\psi\rangle = \alpha|0\rangle + \beta|1\rangle + \gamma|2\rangle \rightarrow |\psi'\rangle = \sqrt{\eta_2}\alpha|0\rangle + (1 - 2\eta_2)\beta|1\rangle - \sqrt{\eta_2}(2 - 3\eta_2)\gamma|2\rangle \quad (19)$$

when  $\eta_1 = \eta_3 = 1$ . There is no solution such that the “0”, “1” and “2” components scale equally, so the gate is biased. However this problem can be solved by placing an additional beamsplitter in the beam path with a vacuum input and conditioning on no photons appearing at its output. Now we find

$$|\psi'\rangle = \sqrt{\eta_2}\alpha|0\rangle + \sqrt{\eta_7}(1 - 2\eta_2)\beta|1\rangle - \eta_7\sqrt{\eta_2}(2 - 3\eta_2)\gamma|2\rangle \quad (20)$$

where  $\eta_7$  is the reflectivity of the additional beamsplitter. Remarkably the additional degree of freedom allows the gate to be rebalanced such that exact NS operation is achieved without an interferometric element. The trade-off is a small reduction in the probability of success. Solving we find  $\eta_2 = (3 - \sqrt{2})/7$  and  $\eta_7 = 5 - 3\sqrt{2}$  gives NS operation with a success probability of  $\eta_2 \approx 0.23$ .

There is considerable flexibility in how the simplified gate is employed in the CNOT. One of a number of possible scenarios is shown in Fig.4. The NS gates of Fig.3 have been replaced by the beamsplitters B5 and B6 which have reflectivities  $\eta_2$ . Additional beamsplitters, B7 and B8, of reflectivities  $\eta_7$  have been inserted in beams  $c_V$  and  $t'$  respectively. The state of the system at point  $z$  in Fig.4 (conditional on a single photon being detected at outputs  $a_{1o}$  and  $a_{2o}$  and no photons appearing at outputs  $v_{7o}$  and  $v_{8o}$ ) is given by

$$|\psi\rangle_y = \frac{1}{\sqrt{2}}\eta_2|1001\rangle \pm \sqrt{\eta_2\eta_7}(1 - 2\eta_2)\frac{1}{2}(|1100\rangle - |1010\rangle) \quad (21)$$

if the control is initially in  $|H\rangle$  and

$$|\psi\rangle_y = \frac{1}{2}(\sqrt{\eta_2\eta_7}(1 - 2\eta_2)(|0101\rangle + |0011\rangle) \mp (\eta_7\eta_2(2 - 3\eta_2)(|0200\rangle - |0020\rangle))) \quad (22)$$

if the control is initially in  $|V\rangle$ . Choosing as before  $\eta_2 = (3 - \sqrt{2})/7$  and  $\eta_7 = 5 - 3\sqrt{2}$  we obtain CNOT operation with a probability  $\eta_2^2 \approx 0.05$ . The operation of the gate can also still be described by Eq.13 but with  $\eta_1 = \eta_3 = 1$  and the substitutions

$$c_V = \frac{\sqrt{\eta_7}}{\sqrt{2}}c'_V + \sqrt{1 - \eta_7}v_8, \quad t' = \frac{\sqrt{\eta_7}}{\sqrt{2}}(t_H + t_V) + \sqrt{1 - \eta_7}v_7 \quad (23)$$

where now  $c'_V$  is the initial state of the control's vertical polarization mode. All the conditional moments of Eq.14-17 are reproduced but with the probabilities of the non-zero moments reduced from 1/16 to approximately 1/20. All other properties of the original gate are retained.

## VI. CONCLUSION

The efficient linear optics computation scheme of Ref. [2] appears exciting in principle but daunting in practice. However we have shown that by adopting a CNOT test architecture the basic principles of the scheme can be tested with present technology. Four-photon experiments with spontaneous sources are difficult, but have been achieved [10]. Basically such experiments utilize events where by chance two down converters simultaneously produce pairs. The use of our simplified scheme would reduce the stability issues in such an experiment significantly with only a small decrease in probability of success. Calculations using Eq.13 show that operation is not critically dependent on experimental parameters. For example 2% errors in beamsplitter ratios only lead to fractions of a percent errors in gate operation.

In the longer term the greater simplicity of our gate is likely to play a significant role in scalable architectures when the required single photon sources and detectors become available.

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## FIGURES

FIG. 1. Review of Teleportation of Gates. (a) shows a basic teleportation circuit. (b) shows two such circuits with a CNOT implemented post teleportation. (c) shows the effect of commuting the CNOT through the  $\sigma_x$  ( $X$ ) and  $\sigma_z$  ( $Z$ ) operations. The dotted line encloses the entanglement resource which could be produced using non-deterministic gates.

FIG. 2. Schematic of NS gate. Grey indicates the surface from which a sign change occurs upon reflection. The use of this beamsplitter phase convention is convenient but not essential.

FIG. 3. Schematic of CNOT gate. Grey indicates the surface from which a sign change occurs upon reflection. Note that if B1 and B4 were not present the gate would implement a control sign shift. B1 and B4 paly the role of Hadamard gates coverting sign shift to CNOT operation

FIG. 4. Schematic of simplified CNOT gate. Grey indicates the surface from which a sign change occurs upon reflection.